Renewable and Sustainable Energy Reviews 38 (2014) 864–875

Contents lists available at ScienceDirect

Renewable and Sustainable Energy Reviews

journal homepage: www.elsevier.com/locate/rser

Lattice Boltzmann method for convective heat transfer of nanofluids – A review

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ARTICLE INFO

Article history:
Received 18 September 2013
Received in revised form
26 April 2014
Accepted 1 July 2014
Available online 26 July 2014

Keywords:
Lattice Boltzmann method
Nanofluid
Nusselt number
Heat transfer

ABSTRACT

In recent years, the lattice Boltzmann method (LBM) has become an alternative and attractive approach to simulate numerous fluid flow problems. A colloidal mixture of nano-sized particles in a base liquid called nanofluid, which is the new generation of heat transfer fluid for various heat transfer applications, has recently been demonstrated to have great potential for improving the heat transfer properties of liquids. This paper intends to provide a brief review of researches on application of lattice Boltzmann method on the prediction of nanofluid and identifies opportunities for future research.

Contents

1. Guideline on how the review was conducted

A comprehensive review of the literature in the area of the lattice Boltzmann method for convective heat transfer of nanofluids published between 2010 and 2013 has been conducted. The journals surveyed in this review include the following: Nanoscale Research Letters, International Communications in Heat and Mass Transfer, International Journal of Thermal Sciences, Powder Technology, Theoretical and Computational Fluid Dynamics, Scientia Iranica, Advances in Mechanical Engineering, Defect and Diffusion Forum, International Journal of Fluid Mechanics Research, and Heat Transfer—Asian Research.

The particular literature would be surveyed and included in this review if the literatures have three characteristics of this review's scopes. The characteristics which the literature must have were (1) numerical, (2) nanofluids, and (3) the lattice Boltzmann method. This review of literature is organized around a topic or issue, rather than the progression of time. However, progression of time is also still an important factor in this review.

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http://dx.doi.org/10.1016/j.rser.2014.07.001
1364-0321/

2. Introduction

For more than a decade, lattice Boltzmann method (LBM) has been demonstrated to be a very effective numerical tool for a broad variety of complex fluid flow phenomena that are problematic for conventional methods [1–19]. Compared with traditional computational fluid dynamics, LBM algorithms are much easier to be implemented especially in complex geometries and multicomponent flows.

Historically, LBM was derived from lattice gas automata (LGA) [3,20]. Consequently, LBM inherits some features from its precursor, the LGA method. The first LBM model was a floating-point version of its LGA counterpart. Each particle in LGA model (represented by single bit Boolean integer) was replaced by a single particle distribution function represented by a floating-point number. The lattice structure and the evolution rule remain the same. One important improvement to enhance the computational efficiency, which has been made for the LBM, was that the linearization of collision operator [21]. The uniform lattice structure remained unchanged.

The starting point in the lattice Boltzmann scheme is by tracking the evolution of the single-particle distribution function. The concept of particle distribution has already been well developed in the field of statistical mechanics while discussing the kinetic theory of gases and liquids [22]. The definition implies the probable number of molecules in a certain volume at a certain time made from a huge number of particles in a system that travels freely, without collision, for long distances (mean free path) compared to their sizes. Once the distribution functions are obtained, the hydrodynamics equations can be derived.

Although the LBM approach treats gases and liquids as systems consisting of individual particles, the primary goal of this approach is to build a bridge between the mesoscopic and macroscopic dynamics, rather than to deal with macroscopic dynamics directly. In other words, the goal is to derive macroscopic equations from mesoscopic dynamics by means of statistic, rather than to solve macroscopic equations.

The LBM has a number of advantages over other conventional computational fluid dynamics approaches. The algorithm is simple and can be implemented with a kernel of just a few hundred lines [23–32]. The algorithm can also be easily modified to allow for the application of other, more complex simulation components. For example, the LBM can be extended to describe the evolution of binary mixtures, or extended to allow for more complex boundary conditions [33–42]. Thus the LBM is an ideal tool in fluid simulation.

3. Lattice Boltzmann formulation

The starting point for lattice Boltzmann simulation is the evolution equation for a set of distribution functions f_i which is discrete in both space and time

$$f_i(\mathbf{x}+\mathbf{e}_i, t+1)-f_i(\mathbf{x}, t)=\frac{1}{\tau_f}\left[f_i(\mathbf{x}, t)-f_i^{eq}(\mathbf{x}, t)\right]+F \tag{1}$$

where $\mathbf{e}$ is the particle's velocity, τ is the relaxation time for the collision, f_i^{eq} is an equilibrium distribution function and $i=0$, 1,…, 8 for two-dimensional nine-velocity model (D2Q9). Noted that the right hand side of Eq. (1) is the collision term where the Bhatnagar–Gross–Krook (BGK) approximation has been applied [39]. The discrete velocity is expressed as $\mathbf{e}_i=(0, 0)$ for $i=0$, $\mathbf{e}_i=(\cos\,(i-1)\pi/4, \sin\,(i-1)\pi/4)$ for $i=1$, 3, 5, 7 and $\mathbf{e}_i=2^{1/2}(\cos\,(i-1)\pi/4, \sin\,(i-1)\pi/4)$ for $i=2$, 4, 6, 8. Macroscopic density ρ and velocity $\mathbf{u}$ of the fluid are determined by the following velocity moments of the distribution function:

$$\sum_i f_i^{eq}=\rho \tag{2}$$

$$\sum_i e_{i,\alpha} f_i^{eq}=\rho u_\alpha \tag{3}$$

The equilibrium distribution function, f_i^{eq}, is chosen such that the continuum macroscopic equations approximated by evolution equation correctly describe the hydrodynamics of the fluid. For D2Q9 model, f_i^{eq} is defined as

$$f_i^{eq}=\rho\omega_i\left[1+3\frac{\mathbf{e}_i\cdot\mathbf{u}}{c^2}+9\frac{(\mathbf{e}_i\cdot\mathbf{u})^2}{2c^4}-\frac{3\mathbf{u}^2}{2c^2}\right] \tag{4}$$

where $c=(3\mathrm{RT})^{1/2}$ and the weights are $\omega_0=4/9$, $\omega_{1,3,5,7}=1/9$ and $\omega_{2,4,6,8}=1/36$. Through multiscaling expansion, the mass and momentum equation can be derived from D2Q9 model as follows:

$$\nabla\cdot\mathbf{u}=0 \tag{5}$$

$$\frac{\partial\mathbf{u}}{\partial t}+\mathbf{u}\nabla\cdot\mathbf{u}=\frac{1}{\rho}\nabla p+\upsilon\nabla^2\mathbf{u} \tag{6}$$

The viscosity, υ, can be related to the time relation in lattice Boltzmann equation as follows:

$$\tau=3\upsilon+\frac{1}{2} \tag{7}$$

4. Lattice Boltzmann for nanofluids simulation

Prediction of thermal field requires a new type of distribution function to represent the evolution of internal energy [43–64]. The most common type of internal energy distribution function is the nine-velocity model. The governing equation is expressed as

$$g_i(\mathbf{x}+\mathbf{e}_i, t+1)-g_i(\mathbf{x}, t)=\frac{1}{\tau_g}\left[g_i(\mathbf{x}, t)-g_i^{eq}(\mathbf{x}, t)\right] \tag{8}$$

The equilibrium distribution function is expressed as

$$g_i^{eq}=\rho T\omega_i\left[1+3\frac{\mathbf{e}_i\cdot\mathbf{u}}{c^2}+9\frac{(\mathbf{e}_i\cdot\mathbf{u})^2}{2c^4}-\frac{3\mathbf{u}^2}{2c^2}\right] \tag{9}$$

The limited heat transfer capabilities of conventional heat transfer liquids such as water, mineral oil and ethylene glycol give an innovative technique to improve the heat transfer. Research conducted by a group at the Argonne National Laboratory was the first to study the use of particles nanometer dimension. Nanofluids, a term proposed by Choi [65] in 1995, is referred to the fluids with suspended nanoparticles having a diameter below 100 nm.

Compared with suspended particles of millimeter-or-micrometer dimensions, which have numerous drawbacks like sedimentation, erosion, fouling and increased pressure drop of the flow channel, the nanofluids suspended in the fluid longer; thus the severity of the obstacles presented reduced by the rapid settling of the particles (such as abrasion and clogging of microchannels and pipes).

Furthermore, heat transfer applications using nanofluids will provide numerous advantages such as miniaturized heat exchangers system, enhanced heat transfer, reduced heat transfer fluid inventory and reduced emissions. Moreover, the reduction of pumping power occurs when the nanofluid conserves the energy. Nanofluids can be used for a wide variety of engineering applications like transportation, electronics, medical, food, defense, nuclear, space, and manufacturing of many types [66].

Basically, nanofluid has different behavior from pure liquid due to interparticle potentials and other forces on the nanoparticles. Therefore, for modeling the nanofluid, some governing equations should be changed because of the changes in the properties such

as thermal conductivity, density, heat capacitance, and thermal expansion [67–70].

The effective density of a nanofluid is

$$\rho_{nf} = (1-\phi)\rho_f + \phi\rho_s \tag{10}$$

The heat capacitance of the nanofluid and part of the Boussinesq term are

$$(\rho c_p)_{nf} = (1-\phi)(\rho c_p)_f + \phi(\rho c_p)_s \tag{11}$$

$$(\rho\beta)_{nf} = (1-\phi)(\rho\beta)_f + \phi(\rho\beta)_s \tag{12}$$

where $\varnothing$ is the volume fraction of the solid particles, subscripts f, nf and s stand for base fluid, nanofluid and solid, respectively.

Even though many models have been developed to predict the nanofluid viscosity and thermal conductivity as shown in Tables 1 and 2, the most common models used in literature were the Brinkman model [71–76] and the Maxwell–Garnetts (MG) model [77–79]:

$$\mu_{nf} = \frac{\mu_f}{(1-\phi)^{2.5}} \tag{13}$$

$$\frac{k_{nf}}{k_f} = \frac{k_s + 2k_f + 2\phi(k_f - k_s)}{k_s + 2k_f - \phi(k_f - k_s)} \tag{14}$$

5. Current application of LBM on nanofluid

Predictions of nanofluid using LBM have been tried by Zhou et al. [103]. They tested the capability of multicomponent and single-components hybrid LBM to investigate the convection process of Cu/water nanofluid in a square cavity.

Few years later, Xuan and You [104] used the lattice Boltzmann model for simulating flow and energy transport process of the

Table 1
Viscosity models for nanofluids.

	Model	Viscosity
1	Einstein [80]	$\mu_{nf} = \mu(1+2.5\phi)\phi < 0.05$
2	Niesen [81]	$\frac{\mu_{nf}}{\mu_f} = (1+1.5\phi_p)e^{\phi_p/(1-\phi_m)}$
3	De Bruijn [82]	$\frac{\mu_{nf}}{\mu_f} = 1+2.5\phi+4.698\phi^2$
44	Mooney [83]	$\frac{\mu_{nf}}{\mu_f} = 1+2.5\phi+[3.125+(2.5/\phi_{max})]\phi^2$
5	Brinkman model [71–76]	$\frac{\mu_{nf}}{\mu_f} = \frac{\mu_f}{(1-\phi)^{2.5}}$
6	Pak and Cho [84]	$\mu_{nf} = \mu_f(1+39.11\phi+533.9\phi^2)$
7	Wang [85]	$\frac{\mu_{nf}}{\mu_f} = 1+7.3\phi+123\phi^2$
8	Maiga [86]	$\frac{\mu_{nf}}{\mu_f} = 123\phi^2+7.3\phi+1$
9	Koo and Kleinstreuer [87]	$\mu_{Brownian} = 5\times10^4\beta\rho m\phi_p\sqrt{\frac{K_BT}{2\rho_p r_p}[(-134.63+1722.3\phi_p)+(0.4705-6.04\phi_p)T]}$
10	Nguyen [88]	$\frac{\mu_{nf}}{\mu_f} = (2.1275-0.0215T+0.00027T^2)$
11	Jang et al. [89]	$\mu_{nf} = \mu_f(1+2.5\phi)\left[1+\eta\frac{d_p}{H}^{-2\varepsilon}\phi^{2/3}(\varepsilon+1)\right]$
12	Brownian model [90]	$\mu_{nf} = \mu_f(1+2.5\phi+6.17\phi^2)$
13	Gherasim [91]	$\frac{\mu_{nf}}{\mu_f} = 0.904e^{14.8\phi}$
14	Chandrasekar [92]	$\frac{\mu_{nf}}{\mu_f} = 1+b\left(\frac{\phi}{1-\phi}\right)^n$

Table 2
Thermal conductivity models for nanofluids.

	Models	Thermal conductivity
1	Patel et al. [93]	$\frac{k_{nf}}{k_f} = 1+\frac{k_p d_f\phi}{k_f d_p(1-\phi)}\left[1+c\frac{2k_BTd_p}{\pi\alpha_f\mu_f d_p^2}\right]$
2	Maxwell–Garnetts (MG) model [77–79]	$\frac{k_{nf}}{k_f} = \frac{k_s+2k_f+2\phi(k_f-k_s)}{k_s+2k_f-\phi(k_f-k_s)}$
3	Chon et al. [94]	$\frac{k_{nf}}{k_f} = 1+64.7\phi^{0.764}\left(\frac{d_f}{d_s}\right)^{0.369}\left(\frac{k_f}{k}\right)^{0.7476}\mathrm{Pr}_T\mathrm{Re}_T^{1.2321}$
4	Wasp [95]	$\frac{k_{nf}}{k_f} = \frac{k_p+2k_f-2\phi(k_f-k_p)}{k_p+2k_f+\phi(k_f-k_p)}$
5	Mintsa et al. [96]	$\frac{k_{nf}}{k_f} = 1.72\phi+1.0$
6	Jang and Choi [97]	$\frac{k_{nf}}{k_f} = (1-\phi)+Bk_p\phi+18\times10^6\frac{3d_f}{d_{nano}}k_f\mathrm{Re}_{d_{nano}}^2\mathrm{Pr}\phi$
7	Bruggeman model [98]	$\frac{k_{nf}}{k_f} = \frac{(3\phi-1)k_p/k_f+[3(1-\phi)-1]+\sqrt{\Delta_B}}{4}$ $\Delta_B = (3\phi-1)k_p/k_f+[3(1-\phi)-1]^2+8k_p/k_f$
8	Koo and Kleinstreuer [87]	$k = k_f\left[\frac{k_p+2k_f-2\phi(k_f-k_p)}{k_s+2k_f-\phi(k_f-k_s)}\right]+5\times10^4\beta\phi\rho_f(C_p)\sqrt{\frac{k_BT}{d_p\rho_p}f(T,\phi)}$
9	Hamilton and Crosser [99]	$\frac{k_{nf}}{k_f} = \frac{k_p+(n-1)k_f-(n-1)(k_f-k_p)\phi}{k_p+(n-1)k_f+(k_f-k_p)\phi}$
10	Charuyakorn et al. [100]	$\frac{k_{nf}}{k_f} = \frac{k_p+2k_f-2\phi(k_f-k_p)}{k_p+2k_f+\phi(k_f-k_p)}(1+b\phi Pe_p^m)$
11	Staionary [101]	$\frac{k_{nf}}{k_f} = \left[1+\frac{k_p\phi d_f}{k_f(1-\phi)d_p}\right]$
12	Yu and Choi [102]	$\frac{k_{nf}}{k_f} = \frac{k_p+2k_f-2\phi(k_f-k_p)(1+\eta)^3}{k_p+2k_f+\phi(k_f-k_p)(1+\eta)^3}$

nanofluid. Some calculations for the distribution of the suspended nanoparticles also have been conducted. They reported that, the distribution of the suspended nanoparticles was determined by a series of acting forces and potentials. Moreover, rising temperature and main flow of the fluid can improve the nanoparticles distribution.

5.1. Natural convection heat transfer

Natural convection heat transfer has long been studied and has received much attention due to its direct relevancy to many engineering applications such as flooding protection for buried pipes, solidification processes, heat exchangers, electronic packaging and chemical reactors. Guiet et al. [105] performed simulation study which used lattice Boltzmann method to investigate the natural convection of nanofluid in shallow cavities. The influence of Cu, Ag and TiO_2 nanoparticles on convectional flows in shallow cavities heated from below was simulated for various Rayleigh numbers. They found a good agreement between the LBM results and the analytical solution provided by Alloui et al. [106].

Kefayati et al. [107] carried out a simulation study to investigate the natural convection of water/SiO_2 nanofluid in an enclosure using the lattice Boltzmann method. The results between LBM and the conventional method have been compared and shown good agreement with each other. They also stated that an increasing trend was observed for average (Nusselt number) Nu as (Rayleigh number) Ra increased from 10^3 up to 10^4 but started to decrease when Ra reached 10^5. The aspect ratios were also reported to be effective on Nu. The findings were consistent with findings of past studies by Lai and Yang [108], who simulated natural convection of Al_2O_3/water nanofluid by performing mathematical modeling in a vertical square enclosure using the lattice Boltzmann method. They found that the Nusselt number increased with the increase of Rayleigh number and particle number concentration. In addition, under the same Rayleigh number, the Nusselt number was higher with the use of nanofluid rather than water. Meanwhile at a fixed temperature, they showed that nanofluid takes on a lower value of heat transfer than water due to the significant enhancement of dynamic viscosity.

In contrast, the study by He et al. [109] developed a lattice Boltzmann model by coupling the density (D2Q9) and the temperature distribution functions with 9-speed to simulate the convection heat transfer utilizing Al_2O_3/water nanofluids in a square cavity indicating that the average Nusselt number is reduced with increasing the volume fraction nanoparticles, especially at high Ra. Apart from that, their results also indicated that the flow and heat transfer characteristics of Al_2O_3/water nanofluid in the square cavity were more sensitive to viscosity than to thermal conductivity.

According to Sajjadi et al. [110] study, the effect of heating the side wall partially and inclination on natural convection flow in a cavity has been analyzed with the lattice Boltzmann method (LBM) using Cu/water nanofluid. Their study has been conducted for different Rayleigh numbers (Ra) 10^3–10^5 and volume fraction changes between 0% and 15% while inclined angle grew from 0º to 60º with interval of 30°. They obtained that the most effect of nanoparticles occurs at $Ra = 10^3$ for various states. Augmentation of inclination angles increases the effect of nanoparticles for various parameters. Also, increment of nanoparticles volume fractions causes enhancement of the value of stream functions and average Nusselt number for different Rayleigh numbers and inclination angles for every case.

In another study, Rahmati et al. [111] carried out a numerical study of heat transfer enhancement in a two-dimensional enclosure by an incompressible generalized lattice Boltzmann method for a range of Grashof numbers and volume fractions. The fluid in the square cavity was a water-based nanofluid containing Cu nanoparticles. They obtained results that the nanoparticles substantially increase the heat transfer rate at any given Grashof number. Besides, they also reported that the nanofluid heat transfer rate enhances with an increase in the nanoparticles volume fraction. Meanwhile, Guiet et al. [112] performed a numerical study on natural convection from a protruding heater located at the bottom of a square cavity filled with a copper–water nanofluid using LBM. They assumed that the heat source was either to be isothermal or to have a constant heat flux. The effect of pertinent parameters such as thermal Rayleigh number, the solid volume fraction of nanoparticles, the size (L and W) of the heater and its position was investigated. Their result for the case of an isothermal heater indicates that, for a given solid volume fraction of nanoparticles, the heat transfer was improved when the size of the heating source increased. Also, the average Nusselt number was enhanced as the position of the heating block is moved from the center position toward the vertical isothermal boundaries of the enclosure.

Furthermore, Guo et al. [113] extended this work to investigate the thermal and flow field of nanofluid natural convection in a square cavity using conventional LBM approach. The heat transfer characteristics of nanofluid were investigated. By comparing heat transfer between nanofluid and base liquid it was found that with the increase of Rayleigh number, the nanofluid thermal boundary layer near the vertical wall turns thin, and the natural convection in the cavity was strengthened and the heat transfer was enhanced. They obtained a correlation for the mean Nusselt number.

Fattahi et al. [114] numerically investigated natural convection flows in a cavity subject to different side wall temperatures. The fluid in the cavity was a water-based nanofluid containing Al_2O_3 and Cu nanoparticles. The effective thermal conductivity and viscosity of nanofluid were calculated using Chon and Brinkman models. The results indicated that by increasing solid volume fraction, the average Nusselt number increased for both nanofluids. Research finding by Kefayati et al. [115] also points towards the same results. In this research, simulations of natural convection in an open enclosure with water/copper nanofluid by the lattice Boltzmann method showed that the average Nusselt number increases with augmentation of Rayleigh number and the volume fraction of nanoparticle for whole ranges of aspect ratios. Also by increasing the aspect ratio at various Rayleigh numbers and changing the nanoparticle volume fractions, the average Nusselt number decreases. They concluded that the most effect of nanoparticles on heat transfer enhancement was at the aspect ratio of $Ar = 2$ and nanoparticles influence the heat transfer less at $Ra = 10^5$.

Recently, Mehrizi [116] investigated the effect of nanoparticles on natural convection heat transfer in two-dimensional horizontal annulus using the lattice Boltzmann method for different Rayleigh numbers and nanoparticles volume fraction. They indicated that Nusselt number increases by an increase of nanoparticle volume fraction. Augmentation of Rayleigh numbers also leads to an increase in Nusselt number. Also, the effect of nanoparticles was more noticeable at low Rayleigh numbers. Results showed that the average Nusselt number increases when the inner cylinder moves downward.

5.2. Mixed convection heat transfer

Meanwhile, a number of studies on mixed convection heat transfer were carried out by some researchers. Early study by Nemati et al. [117] numerically investigated the mixed convection flows utilizing nanofluids in a lid-driven cavity using the lattice Boltzmann method. The fluid in the cavity was a water-based nanofluid containing Cu, CuO or Al_2O_3 nanoparticles. The effects of Reynolds number and solid volume fraction for different nanofluids on hydrodynamic and thermal characteristics were investigated. The results indicated

Table 3
Summary of the lattice Boltzmann method studies on nanofluid.

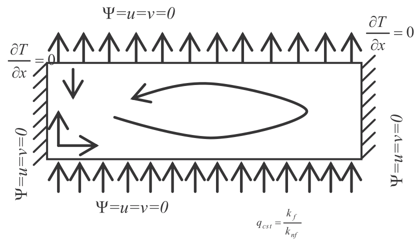
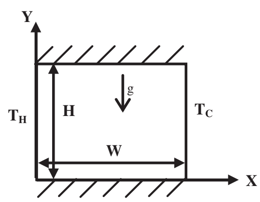
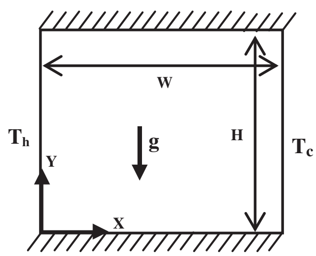
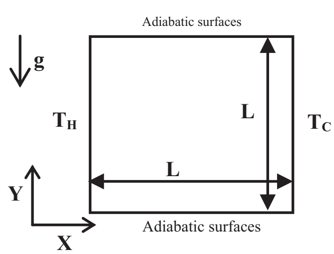
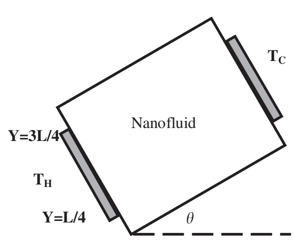
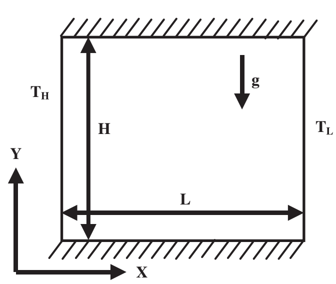
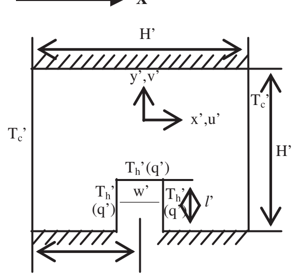
	Authors	Configuration	Nanofluid	Nanofluid fraction
1	Guiet et al. [105]		Cu/Ag /TiO_2- water	0,0.1,0.2
2	Kefayati et al. [107]		SiO2-water	0,0.01,0.02,0.03,0.04
3	Lai et al. [108]		Al_2O_3-water	0,0.01,0.02,0.03,0.04
4	He et al. [109]		Al_2O_3-water	0,0.01,0.03,0.05
5	Sajjadi et al. [110]		Cu-water	0,0.05,0.1,0.15
6	Rahmati et al. [111]		Cu-water	0,0.08,0.16,0.20
7	Guiet et al. [112]		Cu-water	0,0.01,0.02,0.03,0.04,0.05

Table 3 (*continued*)

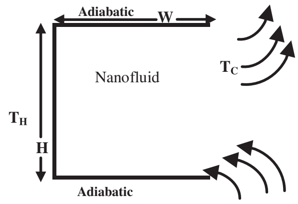
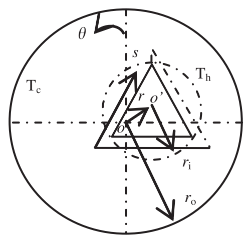
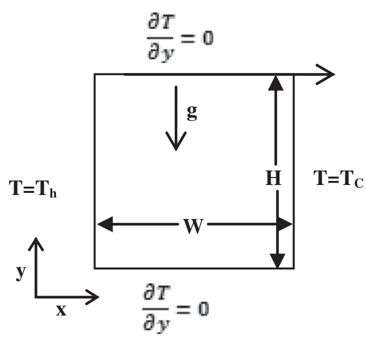
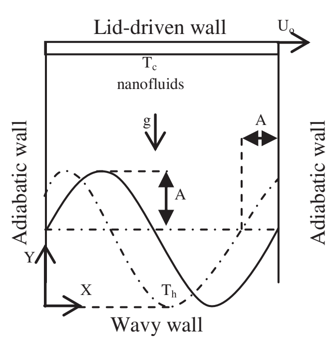
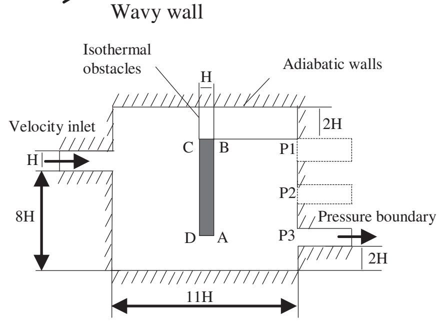
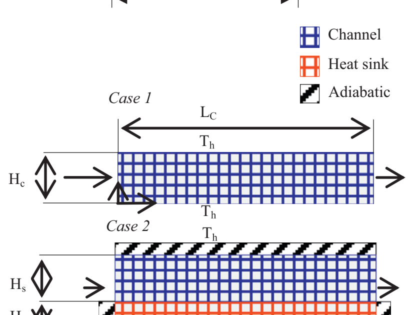
	Authors	Configuration	Nanofluid	Nanofluid fraction
8	Guo et al. [113]		Al_2O_3-water	0,0.005,0.01,0.05,0.1,0.15
9	Fattahi et al. [114]		Al_2O_3/Cu-water	0,0.01,0.02,0.03,0.04,0.05
10	Kefayati et al. [115]		Cu-water	0,0.01,0.02,0.03,0.04,0.05
11	Mehrizi et al. [116]		Cu–Water	0,0.05,0.1
12	Nemati et al. [117]		Cu/CuO/Al_2O_3-water	0.0.01,0.03,0.05
13	Jafari et al. [118]			0,0.01,0.02,0.03,0.04,0.05
14	Mehrizi et al. [119]		Cu-water	0,0.01,0.02,0.03
15	Yang et al. [120]		Al_2O_3-water	0,0.01,0.02,0.03,0.04

Table 3 (*continued*)

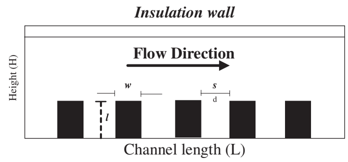
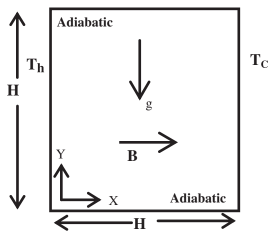
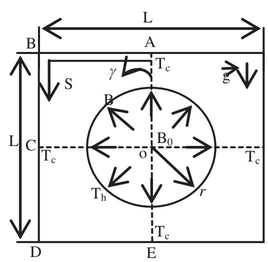
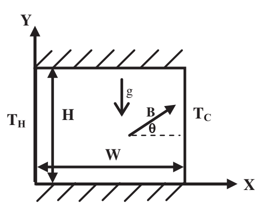
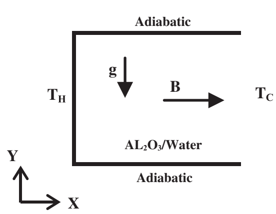
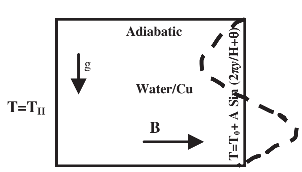
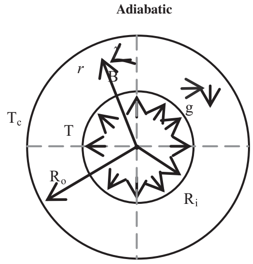
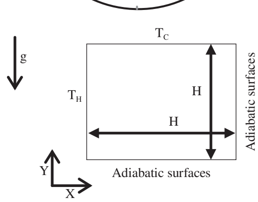
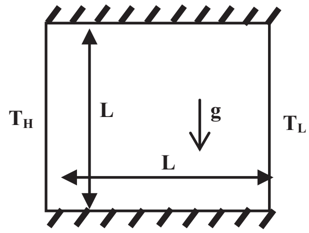
	Authors	Configuration	Nanofluid	Nanofluid fraction
16	Azwadi et al. [121]		Al_2O_3-water	0,0.03,0.05
17	Nemati et al. [122]		CuO-water	0,0.01,0.02,0.03,0.04,0.05
18	Sheikholeslami et al. [123]		Cu-water	0, 0.02,0.04 and 0.06
19	Kefayati [124]		Cu-water	0,0.01,0.02,0.03,0.04, 0.05,0.06
20	Kefayati [125]		Al_2O_3-water	0,0.02,0.04,0.06
21	Kefayati [126]		Cu-water	0,0.01,0.02,0.03,0.04,0.05,0.06
22	Ashorynejad [127]		Ag-water	0, 0.02,0.0 4,0.06
23	Qi et al. [128]		Al_2O_3- Water	0,0.01,0.03,0.05

Table 3 (*continued*)

	Authors	Configuration	Nanofluid	Nanofluid fraction
24	Nabavitabatabayi et al. [129]		Cu/Ag/ Al_2O_3 /TiO_2- water	0,0.05,0.1,0.15

that the effects of solid volume fraction grow stronger sequentially for Al_2O_3, CuO and Cu. Besides the increases of Reynolds number led to decrease in the solid concentration effect.

Concurrently, Jafari et al. [118] numerically investigated the effect of Cu and CuO nanoparticles' presence on mixed convection heat transfer in a lid-driven cavity with a corrugated wall using the boundary fitting method in the lattice Boltzmann method. The effect of the volume fraction of the nanoparticles on the local Nusselt number distribution, average Nusselt number, streamlines, and temperature contours was investigated for various Richardson numbers, when the Reynolds number was fixed to 25. It was found by the results that nanoparticles have significant effects on both fluid flow and heat transfer of the mixed convection, especially for low Richardson numbers. It was also observed that increasing the wavy wall's amplitude leads to a decrease of the average Nusselt number for a high Richardson number.

Another numerical study was performed by Mehrizi et al. [119] to investigate the effect of suspension of nanoparticles on mixed convection in a square cavity with inlet and outlet ports and hot obstacle in the center of the cavity using LBM. The effect of outlet port location on heat transfer rate and the effect of nanoparticles were studied. They observed that the maximum heat transfer rate occurs when the outlet port was located at P2 for Ri = 0.1 and P1 for Ri = 1, Ri = 10. The results showed that by adding the nanoparticles to base fluid and increasing the volume concentration of nanoparticles the heat transfer rate was enhanced at different Richardson numbers and outlet port positions. Table 3 shows the summary of the published lattice Boltzmann method investigations of the convective heat transfer performance on various nanofluids.

5.3. Forced convection heat transfer

Yang and Lai [120] used the lattice Boltzmann method to simulate forced convection flow of Al_2O_3/water nanofluids in a microchannel by mathematical modeling. Their results showed that, average Nusselt number increases with the increase of Reynolds number and particle volume concentration. The fluid temperature distribution was more uniform with the use of nanofluid than that of pure water. Furthermore, the authors compared the results with those achieved from finite volume based numerical methods and reported a good agreement implying LBM's ability to be applied to various engineering problems.

In a different study, Nor Azwadi et al. [121] presented a numerical study of the thermal performance of fins mounted on the bottom wall of horizontal channel using the lattice Boltzmann method. They examined the heat transfer performance by adding the fin and also using the nanofluid. Reynolds number and nanoparticle volume fraction varied from 10 to 100 and from 0 to 0.05 in their study, respectively. Their result showed that the heat transfer rate of fins was significantly affected by the Reynolds number and the thermal conductivity of the fins. They also highlight that the influence of the solid volume fraction on the increase of heat transfer is more noticeable at higher values of Re.

5.4. Effect of magnetic field

Nemati et al. [122] conducted a numerical investigation of CuO nanoparticles on natural convection with magnetohydrodynamic (MHD) flow in a square cavity using LBM. They investigated the effects of the solid volume fraction and magnetic field on hydrodynamic and thermal characteristics. They obtained that the magnetic field reduces the circulation in the cavity. When the magnetic field becomes stronger, it causes the convection heat transfer to reduce and, subsequently, conduction heat transfer becomes dominant. Therefore, by adding nanoparticles to fluid, the average Nusselt number increases, and by increasing the volume fraction, the phenomenon becomes more sensible. It was also obtained that the lattice Boltzmann method, based on a multi-distribution function, was a powerful approach for simulating nanofluid flow in the presence of a magnetic field.

The above finding is consistent with the study by Sheikholeslami et al. [123]. Author numerically investigated the natural convection in a concentric annulus between a cold outer square and heated inner circular cylinders in the presence of static radial magnetic field using the lattice Boltzmann method. They carried out investigation for different governing parameters namely the Hartmann number, nanoparticles volume fraction and Rayleigh number. It was revealed that the average Nusselt number was an increasing function of nanoparticle volume fraction as well as the Rayleigh number, while it was a decreasing function of the Hartmann number. In addition, it can be found that for Hartmann number (Ha) > 20 the enhancement in heat transfer at $Ra = 10^4$ is greater than at other Rayleigh numbers. Also, for Ha > 20, maximum heat transfer enhancement was obtained at $Ra = 10^5$.

In the meantime, Kefayati [124] analyzed the effects of a magnetic field on natural convection flow in filled long enclosures with Cu/water nanofluid by using the lattice Boltzmann method. They carried out investigation for the pertinent parameters in the following ranges: the Rayleigh number of base fluid, $Ra = 10^3–10^5$; the volumetric fraction of nanoparticles between 0% and 6%; and the aspect ratio of the enclosure between $A = 0.5$ and 2. Results showed that the heat transfer decreases by the increment of Hartmann number for various Rayleigh numbers and the aspect ratios. Also, heat transfer decreases with the growth of the aspect ratio but this growth causes the effect of the nanoparticles to increase. In addition, the magnetic field augments the effect of the nanoparticles at high Rayleigh numbers ($Ra = 10^5$).

Recently, Kefayati et al. [125] conducted a research for effect of a magnetic field on natural convection in an open enclosure which subjugated to water/copper nanofluid using the lattice Boltzmann Method. It emerged the heat transfer decreases by the increment of Hartmann number for various Rayleigh numbers and volume fractions. In addition, they mentioned the magnetic

field augments the effect of nanoparticles at Rayleigh number of $Ra=10^6$ regularly.

In another studies, Kefayati [126] performed a numerical study to examine the effect of a magnetic field on natural convection flow in a nanofluid-filled cavity with sinusoidal temperature distribution on one side wall with a new attitude to the lattice Boltzmann method (LBM). The cavity was filled with water and nanoparticles of Cu in the presence of a magnetic field. They studied the pertinent parameters in the following ranges: Rayleigh number of the base fluid, $Ra=10^3–10^5$; the volumetric fraction of the nanoparticles between 0% and 6%; phase deviation ($\theta=0$, $\pi/4$, $\pi/2$, $3\pi/4$, and π); and Hartmann number varied from $Ha=0$ to 90 with interval 30 while the magnetic field is considered horizontal. Results showed that the heat transfer decreases by the increment of Hartmann number for various Rayleigh numbers. In this research, the following formulation of external force has been applied to apply the magnetic field effect:

$$F=F_x+F_y \tag{15}$$

$$F_x=3\omega_K\rho(A(\upsilon\ \sin(\gamma)\cos(\gamma))-(u\sin^2(\gamma))) \tag{16}$$

$$F_y=3\omega_k\rho((g\beta(T-T_m))+A(u\ \sin(\gamma)\cos(\gamma))-(\upsilon\cos^2(\gamma))) \tag{17}$$

where A was defined by

$$A=(\mathrm{Ha}^2)\left(\frac{\vartheta}{M^2}\right) \tag{18}$$

The numerical results of natural convection heat transfer in a horizontal cylindrical annulus enclosure filled with nanofluid using the lattice Boltzmann method (LBM) were carried out by Ashorynejad [127]. The simulations were performed for different governing parameters namely, Hartmann number, nanoparticle volume fraction and Rayleigh number. They found out that the absolute values of stream function increases as nanoparticle volume fraction or Rayleigh number increases, while it decreases as Hartmann number increases. It was also found that average Nusselt number was an increasing function of nanoparticle volume fraction and Rayleigh number, while it was a decreasing function of Hartmann number. In this research, the following equation has been modified from the distribution function equation to consider the magnetic effect:

Density distribution functions (magnetic effect)

$$f_i^{eq}=w_i p\left[1+\frac{c_i u}{c_s^2}+\frac{1}{2}\frac{(c_i u)^2}{c_s^4}-\frac{1}{2}\frac{u^2}{c_s^2}\right]+\frac{w_i}{2c_s^2}\left[\frac{B^2c^2}{2}-(cB)^2\right] \tag{19}$$

$$g_i^{eq}=w_i T\left[1+\frac{c_i u}{c_s^2}\right] \tag{20}$$

Magnetic equilibrium function

$$h_{ix}^{eq}=Q_i\left[B_x+\frac{1}{c_s^2}e_{ix}(u_yB_x-u_xB_y)\right] \tag{21}$$

$$h_{iy}^{eq}=Q_i\left[B_y+\frac{1}{c_s^2}e_{iy}(u_xB_y-u_yB_x)\right] \tag{22}$$

More recently, He et al. [128] focused on the study of the natural convection of a square enclosure filled with Al_2O_3 nanofluid using two-phase lattice Boltzmann method. The simulation was performed for different nanoparticle fractions, different Rayleigh numbers, the effects of forces on the nanoparticles volume fraction distribution and the heat transfer. They found that the temperature difference driving force was the biggest force and has the greatest effect on nanoparticle volume fraction distribution. Furthermore, for a low Rayleigh number the nanoparticle volume fraction was low in the lower right corner and high in the top right corner and lower left corner. In this research, the following formulation of external force has been applied to apply the magnetic field effect.

The gravity and buoyancy force:

$$F_H=-\frac{4\pi\alpha^3}{3}g\Delta p' \tag{23}$$

The drag force (Stokes force):

$$F_D=-6\pi\mu\alpha\Delta u \tag{24}$$

The interaction potential force:

$$F_A=\sum_{i=1}^{8}n_i\frac{\partial V_A}{\partial r_i} \tag{25}$$

The Brownian force:

$$F_B=G_i\sqrt{\frac{c}{dt}} \tag{26}$$

Total per unit volume forces acting on nanoparticles of a single lattice:

$$F_P=n(F_H+F_D+F_A+F_B)/V \tag{27}$$

Force acting on the base fluid:

$$F_w=-n(F_D+F_B) \tag{28}$$

5.5. Other

Nabavitabatabayi et al. [129] numerically studied the heat transfer performance in an enclosure including nanofluids with a localized heat source. This study used Multiple Relaxation time lattice Boltzmann modeling (MRT) which has superior numerical advantages to single-relaxation-time lattice Boltzmann method. They found that the key factor of changes in the heat transfer mechanism was the Rayleigh number which dominates the flow field. As the Rayleigh number increases, the consequent enhanced convective properties cause the maximum temperature to be lowered.

6. Conclusion

The growing interest in using the LBM to predict the enhancement or control of heat transfer using nanofluid has led to significant progress in this method in recent years. This review presented the recent studies on heat transfer enhancement in natural, forced and mixed convection with nanofluid using LBM. The results from such simulations have also shown promising applications in systems especially with magnetic field effects. Nanofluids can be considered to be the next-generation heat transfer fluids as they offer exciting new possibilities to enhance heat transfer performance compared to pure liquids. However, research of nanofluid using LBM still at infant stage. More numerical works need to be done in the future using LBM to compare its validity with the conventional methods. Therefore, several important concerns have been drawn which should receive attention in future. Concerning the numerical studies which try to explain the observed anomalous enhancement in heat transfer, it will be necessary to consider not only one single phase model but two-phase approach, which seems a better model to describe the nanofluid flow since slip velocity between the particle and base fluid plays an important role on the heat transfer performance of nanofluids. Furthermore, future numerical studies can be performed with the effect of different geometry and fluid flow regions especially in turbulent region. Additionally, future numerical studies could be directed toward radiation performance or boiling

heat transfer of nanofluids due to lack of studies on that area. The outcomes from this literature are summarized as follow:

- It is found that heat transfer enhancement with nanofluids using the Lattice Boltzmann method is in good agreement with other numerical methods and experimental results.
- It was observed from the literature that previous researches only focus on laminar flow and only for a square and cylindrical geometry.
- It has been found that Al_2O_3 and Cu were the type of nanofluids used most in the literature. It is also been found that the effects of solid volume fraction for Cu are stronger than Al_2O_3.
- It was also found that Nusselt number is an increasing function of nanoparticle volume fraction, Rayleigh number and Reynolds number, while it is a decreasing function of Hartmann number.

Acknowledgments

The authors would like to acknowledge Universiti Teknologi Malaysia and Ministry of Higher Education, Malaysia (Grant number 00H23) for supporting this research activity.

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